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New Features of Finance Theory

Nontraded assets and the CAPM

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Abstract

Asset pricing models that rely on the presence of non-tradable assets (such as human wealth) to solve the equity premium puzzle have to confront the effect of decreasing absolute risk aversion: rich investors, who according to micro data hold the stock market and whose behavior is the one that matters, at the margin, for the determination of equilibrium asset prices, are less risk averse, ceteris paribus, than the average consumer. This paper highlights a channel through which the effect of decreasing absolute risk aversion can be overcome: the existence of a positive correlation between the rates of return on traded assets and on the human capital of marginal investors.

Key words: Nontraded assets; CAPM; Incomplete markets; Human capital

JEL classification: G1; E6

1. Introduction

The asset pricing literature has been moving away in recent years from the complete markets, Arrow–Debreu paradigm that it had used in previous years. There are several (good) reasons for this trend:

- the inability of complete-market fixes to the basic Mehra and Prescott (1985) framework to deliver convincing accounts of financial data;
- the evidence[1] that, contrary to what should happen under complete markets, intertemporal marginal rates of substitution are not perfectly correlated between consumers;

[1] See, most notably, Cochrane (1988).

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– the fact that stocks are, by and large, only held by wealthy consumers with large labor incomes,[2] contrary to what representative agent models would imply.

The picture that emerges from the data is thus that the marginal investors who hold the stock market are quite distinct, both in terms of asset holdings and consumption pattern, from the 'average' consumer hypothesized by representative agent models. But, in abandoning the complete market paradigm, theory has to address the question of whether a model in which marginal investors with large non-traded incomes hold the stock market can replicate financial data.

In this paper, I wish to highlight one fundamental difficulty that non-representative agent models of asset prices have to solve if they are to be empirically successful: decreasing absolute risk aversion. If the stock market is held by marginal investors who receive large non-traded income, then, in the absence of any countervailing effect, decreasing absolute risk aversion – an attribute that I take to be a fundamental characteristic of preferences – makes marginal investors less risk averse than the average consumer, and thus lowers the equity premium relative to the already much too low level predicted by the representative agent, or 'average' agent, model. Using a simple extension of the static CAPM to allow for non-marketable assets, I study a channel through which the effect of decreasing absolute risk aversion can be overcome: the existence of a positive correlation between the rates of return on traded assets and on the human capital of marginal investors.

This paper is closely related to earlier work by Mayers (1972).[3] Mayers showed how to generalize the *static* capital asset pricing model to account for the presence of nontraded assets. He used a mean–variance framework, and he did not focus on the effect of the size of human wealth relative to total wealth. By contrast, this paper (i) assumes that consumers have constant relative risk aversion and uses log-linear approximations of the budget constraint à la Campbell and Shiller (1988), and (ii) it characterizes carefully the impact on portfolio choice of the fact that almost two-thirds of wealth is held in non-marketable form.

This paper is also related to recent seminal work by Svensson and Werner (1993) and Koo (1991), who have investigated consumption and portfolio choice in the presence of nontraded income risk in a *dynamic* stetting. Unfortunately, their results do not yield analytical solutions for power utility functions (that exhibit the desirable property of decreasing absolute risk aversion), because of the unavoidable difficulty involved with the computation of the present value of a non-traded income stream when absolute

[2] See Mankiw and Zeldes (1989).

[3] I am grateful to Bernard Dumas for pointing out this reference to me at the publishing stage of this paper.

aversion to risk varies with the level of wealth. Because I wish to use, for expository reasons, analytical solutions rather than numerical simulations, and because I view the assumption of decreasing absolute risk aversion as more crucial to the issues I want to study than that of a dynamic model, I couch this paper in essentially static terms, and assume that consumers have power utility functions.

The paper is organized as follows. Section 2 builds a simple static portfolio choice model. This model is solved in Section 3 using a log-linear approximation of the budget constraint, and used in Section 4 to highlight the role of decreasing absolute risk aversion on equilibrium asset prices in a non-representative agent setting. The conclusion summarizes the results.

2. A simple model

In this paper, I use the simplest model that enables me to make the points that this paper seeks to address: a two-period model in which agents consume only in the second period of their life.

As usual, the two-period assumption can be interpreted in either one of two ways:

- The first period represents the present, while what we call the second period subsumes the future.
- Alternatively, the timing of the model can be taken literally, and one can instead imagine that there are factors (e.g., very large costs of transacting with the future) that shorten the horizon of consumers to two periods.

Depending on the view one adopts (they are not exclusive from one another), a 'large' second-period shock can be thought of as either a permanent shock, or a temporary shock in the presence of transactions costs.

The assumption that agents consume only in the final period of their life is a standard simplification in the finance literature. It is not innocuous, however, as it postulates a 100% propensity to save in the first period. Such a setup precludes by construction any analysis of the effects of 'prudence' (as defined by Kimball (1990)) on portfolio choice, but it simplifies the model considerably. I shall return to the role of prudence below.

Consumers are endowed in the first period with total wealth W, a *given* fraction θ of which is held in tradable form and $1-\theta$ in non-tradable form. I will often refer for simplicity to tradable wealth as financial wealth, and to non-tradable wealth as human wealth, but these labels are only illustrative: what matters is marketability or absence thereof. Letting R_W denote the gross rate of return on total wealth, R_M the gross rate of return on marketable wealth, and R_H the gross rate of return on human wealth, consumption in the second period is given by

$$C = R_W W, \tag{2.1}$$

where, by definition,

$$R_W = \theta R_M + (1-\theta) R_H. \tag{2.2}$$

Finally, assume that marketable wealth can be invested in a risky asset offering a gross return R_1, or in riskless asset offering a gross return R_0. Then, if a consumer invests a fraction α of his tradable wealth in the risky asset, the rate of return on his tradable wealth will be

$$R_M = \alpha R_1 + (1-\alpha) R_0. \tag{2.3}$$

Substituting (2.2) and (2.3) into (2.1), we have

$$C = \{\theta[\alpha R_1 + (1-\alpha) R_0] + (1-\theta) R_H\} W. \tag{2.4}$$

The rates of return R_0, R_1 and R_H are assumed to be given by technology. Note that the R_H's and the θ's may in principle differ across individuals.

Because my main focus is on capital market imperfections and not preferences, I assume that all consumers have an increasing and concave utility function

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma},$$

and are expected utility maximizers. The assumption that utility exhibits constant relative risk aversion γ is done to ensure that consumers have decreasing absolute risk aversion, and that, as a consequence, their willingness to hold risky assets depends on the level of their non-traded wealth. While the alternative assumption of constant absolute risk aversion would be analytically simpler and would enable me to deal easily with a truly dynamical framework,[4] its cost would be prohibitive in terms of substance, as constant absolute risk aversion evacuates by construction the very effects, that depend on *decreasing* absolute risk aversion, that I wish to study.

By definition, the allocation of a consumer's wealth between traded and non-traded assets (as governed by θ) is not a choice variable. But the composition α of the portfolio of traded asset is, and, as a consequence, a consumer's objective is to choose a portfolio to maximize the expected utility of consumption:

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[4] It would suffice to use the analytical results in Svensson and Werner (1993).

$$\max \mathrm{E}u(C), \tag{2.5}$$

subject to the budget constraint (2.4) and to the additional constraint $C \geqslant 0$. No short sales constraint is imposed.

The first-order condition for an interior maximum is simply

$$\mathrm{E}[R_1 u'(C)] = R_0 \mathrm{E}u'(C). \tag{2.6}$$

3. An approximate solution

Denote by lower case letters the logarithm of the variable denoted by an uppercase letter, and use the approximation (which is more accurate the closer to zero the mean and the variance of X)

$$x \equiv \log X \approx X - 1. \tag{3.1}$$

Substituting into Eqs. (2.2) and (2.3), we find that

$$r_W \approx \theta r_M + (1-\theta) r_H, \tag{3.2}$$

$$r_M \approx \alpha r_1 + (1-\alpha) r_0. \tag{3.3}$$

Using (2.1), these equations imply that

$$c \approx w + \theta[\alpha r_1 + (1-\alpha) r_0] + (1-\theta) r_H. \tag{3.4}$$

Assume further that r_1 and r_H are jointly normal, with variance–covariance matrix

$$\begin{pmatrix} \sigma_{1H} & \sigma_{1H} \\ \sigma_{1H} & \sigma_{HH} \end{pmatrix}$$

Using the log-linearized budget constraints (3.2) and (3.3), the first-order condition (2.6) can then be rewritten as

$$\mathrm{E}r_1 - r_0 = -\tfrac{1}{2}\sigma_{11} + \gamma\sigma_{1C}, \tag{3.5}$$

where σ_{1C} denotes the covariance between the (log) rate of return on the risky traded asset and (log) consumption. Not surprisingly, the excess return on the risky traded asset is (up to a Jensen's inequality term) proportional to the covariance between the return on the risky traded asset and consumption, with the coefficient of proportionality equal to the coefficient of relative risk aversion γ. Since consumption is perfectly correlated with the rate of return on wealth in this static framework [see (2.1)], Eq. (3.5) is analogous to a static CAPM.

Now, from Eq. (3.4),

$$\sigma_{1C}=\theta\alpha\sigma_{11}+(1-\theta)\sigma_{1H}. \tag{3.6}$$

Substituting into Eq. (3.5) and solving for the fraction α of tradable wealth held in the risky asset, we find

$$\alpha=\frac{1}{\theta}\left[\frac{Er_1-r_0}{\gamma\sigma_{11}}+\frac{1}{2\gamma}\right]+\left(1-\frac{1}{\theta}\right)\frac{\sigma_{1H}}{\sigma_{11}}. \tag{3.7}$$

To interpret (3.7), it is best to three important special cases:

A. *When all wealth tradable* ($\theta=1$), the fraction of tradable wealth held in the risky asset is given by the term in square brackets:[5] up to the Jensen's inequality term $1/(2\gamma)$, this expression is the standard formula for the share of wealth invested in the risky asset when all assets are traded.

B. *When the rates of return on traded and non-traded assets are uncorrelated* ($\sigma_{1H}=0$), the larger the fraction of wealth held in non-tradable form (i.e., the smaller θ), the larger the fraction of tradable wealth invested in the risky tradable asset. In other terms, forcing an investor to hold a larger fraction of her wealth in a non-traded asset leads her to increase the share in her portfolio of any asset whose return is uncorrelated with the return on the non-traded asset. That is, an investor is willing to bear more risk in financial markets if she can only invest a small fraction of her total wealth in these markets – provided, however, returns on traded and non-traded assets are uncorrelated. This effect is due to decreasing absolute risk aversion: a large fraction of wealth held in non-marketable, human form makes uncorrelated financial risks appear less unpalatable to investors who have decreasing absolute aversion to risk.[6]

C. *When the traded and non-traded risky returns are positively correlated* ($\sigma_{1H}>0$), the share of the tradable risky asset in the portfolio might decrease when θ declines provided that σ_{1H}/σ_{11} is sufficiently large. This is due to the fact that a positive covariance between traded and non-traded risky returns reduces the willingness of investors to hold the risky traded asset, as it tends to increase the covariance between the traded risky return and consumption. Whether this effect overcomes, when θ declines, the opposite effect highlighted in special case B depends on how large σ_{1H} is.

[5] In what follows, I assume that $Er_1>r_0$.

[6] As it should, this effect would vanish were we to assume that the utility function is exponential.

4. Implications for asset pricing puzzles

We can now address the question asked in the introduction: how likely is it that a model in which marginal investors with large non-traded incomes hold the stock market replicates financial data?

Provided we are willing, for simplicity, to make the assumption that there is a *representative marginal investor*, the bare-bones model of the previous section provides us with some elements of an answer.[7]

Our analysis of special case B has shown that the presence of non-traded assets, however, poses a fundamental difficulty to the incomplete market explanation of the equity premium puzzle. If the rates of return on traded and non-traded assets are uncorrelated, the larger the fraction of wealth held in non-tradable form, the *more* willing to hold stocks marginal investors should be. In an equilibrium model in which asset prices are determined by the behavior of marginal consumers, this effect would *lower* the equity premium relative to that predicted by a representative agent model – hardly a success.

To illustrate the quantitative importance of this effect, it suffices to make some simple calculations. First rewrite (3.7) as

$$\mathrm{E}r_1 - r_0 + \tfrac{1}{2}\sigma_{11} = \gamma[\theta\alpha\sigma_{11} + (1-\theta)\sigma_{1H}]. \tag{4.1}$$

Assume, as in Lucas (1978) or Mehra and Prescott (1985), that all bonds are inside bonds, so that the equilibrium share of the risky asset in financial wealth, α, is 100% in equilibrium. Equilibrium returns should then satisfy

$$\mathrm{E}r_1 - r_0 + \tfrac{1}{2}\sigma_{11} = \gamma[\theta\sigma_{11} + (1-\theta)\sigma_{1H}]. \tag{4.2}$$

According to the evidence gathered by Mehra and Prescott (1985), the premium of equity over riskless bonds (the sum of the three terms on the left-hand side of (4.2)) has been approximately 6% per year over the last century, while the volatility of stock returns has historically been about 3% per year. As is well known, the standard static CAPM, which neglects the effect of non-traded assets, does quite well in explaining these data: if all assets are traded ($\theta = 1$), then a (quite plausible) coefficient of relative risk aversion γ equal to 2 is all we need to get the left- and right-hand sides of (4.2) to match up.[8]

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[7] In a more general model that would allow for non-degenerate saving behavior, another element, as we shall see below, would be 'prudence', i.e., the fact that portfolio choice is affected by precautionary savings motives.

[8] This 'success' is due to the use of a static framework. In a dynamic framework, however, we know from Mehra and Prescott (1985) that a much higher γ would be needed.

The trouble starts, however, as soon as we allow for a large fraction of total wealth to be non-tradable. Suppose, for instance, that θ is 1/3 (the share of labor income in total output, $1-\theta$, is about 2/3). Then, if traded and non-traded returns are uncorrelated (special case B), i.e. if $\sigma_{iH}=0$, the equity premium predicted by the model if we stick to a value of $\gamma=2$ is now just 1%, i.e. a third of is predicted by the model that (wrongly) assumes that all wealth is marketable. To put it differently but equivalently, we now need $\gamma=6$, a value three times larger, to reconcile the left- and right-hand sides of (4.2).

To overcome the effect of decreasing absolute risk aversion, we need, as noted in special case C, a positive σ_{iH}. But how big a positive correlation is required in practice to overcome the effect of decreasing absolute risk aversion? It is obvious that if we want to reconcile the left- and right-hand sides of (4.2) while maintaining the assumption that $\gamma=2$, we need $\sigma_{iH}=3\%$ per year. It is unfortunately very hard to ascertain whether this value is too large to be borne out by the data. From the aggregate evidence gathered by Fama and Schwert (1976), the correlation between the stock market and aggregate labor income is very small, but it can be large for individual assets.[9] Presumably, too, this low aggregate correlation is consistent with a high correlation between the stock market and the labor income of some individuals, and in particular with a high correlation between the stock market and the labor income of the marginal investors (think of incentive schemes that tie the labor income of Wall Street participants to the performance of the stock market).

Thus, the empirical case for the non-marketable assets model cannot be adjudicated solely on the basis of aggregate evidence. What the theoretical exercise carried out in this paper shows, however, is that the incomplete market model has to overcome the obstacle of decreasing absolute risk aversion if it is to be consistent with data that show that marginal investors are wealthy and thus, ceteris paribus not very risk averse. One channel through which this effect can be overcome has been highlighted here: the necessity of a large and positive correlation between the rate of return on traded assets and the rate of return on the human capital of marginal investors. Another channel, that I have not explored here but analyzed in Weil (1992), is related to 'prudence': when preferences exhibit both relative risk aversion and decreasing absolute prudence, and when consumers care about both consumption in both periods, the existence of uninsurable labor income risk uncorrelated with the return on the financial market portfolio

[9] Fama and Schwert estimate the regression coefficient of the rate of growth of aggregate per capita labor income on the return of the value-weighted portfolio of NYSE stocks to be slightly less than 2%. This is an indication that σ_{1H}/σ_{11}, which this regression coefficient attempts to measure, is quite small in aggregate data.

unambiguously raises, for a given mean labor income, the equilibrium premium on equity over bonds by making consumers more reluctant to hold the stock market.[10]

5. Conclusion

Asset pricing models that rely on the presence of non-tradable assets (such as human wealth) to solve the equity premium puzzle have to confront the effect of decreasing absolute risk aversion: rich investors, who according to micro data hold the stock market and whose behavior is the one that matters, at the margin, for the determination of equilibrium asset prices, are less risk averse, ceteris paribus, than the average consumer. This paper highlights a channel through which the effect of decreasing absolute risk aversion can be overcome: the existence of a positive correlation between the rates of return on traded assets and on the human capital of marginal investors.

[10] This effect is absent by construction from the essentially static model studied here as individuals do not care by assumption about first-period consumption, and thus have the same saving behavior regardless of the uncertainty they are facing in the second period. The model in Weil (1992), however, cannot be used to analyze the issues studied here, as it is a model in which all agents are identical ex ante.

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