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OVERLAPPING FAMILIES OF INFINITELY-LIVED AGENTS

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This paper develops a model in which new and infinitely-linked dynasties, which are, by definition, not linked to pre-existing families through operative intergenerational transfers, continuously enter the economy over time. This model of infinitely-lived families possesses most of the properties characteristic of standard overlapping generation frameworks: competitive equilibria can be inefficient, bubbles may exist, and Ricardian neutrality does not in general hold. Contrary to a widespread but erroneous belief, the terms 'infinite horizon model' and 'representative agent model' are, therefore, not interchangeable.

1. Introduction

It is sometimes argued that the Ricardian debt neutrality proposition holds whenever the effective length of the consumers' planning horizon is infinite;[1] or that the existence of asset bubbles is ruled out by the transversality constraints associated with the optimal consumption program of infinitely-lived consumers.[2]

Yet we know that the validity of the Ricardian neutrality debt proposition depends on the identity of the taxpayers[3] and the existence of a representative agent, and not on the length of horizons; and that it is a zero-sum game argument[4] which rules out the existence of bubbles in economies with a fixed number of identical agents, and not one based on infinite lives.

Therefore, the terms 'infinite horizon model' and 'representative agent model' should not be used interchangeably – as they unfortunately so often are in macroeconomics. The length of the consumers' planning horizons bears no logical relation to the issues raised by the Ricardian debt neutrality proposition, the existence of asset bubbles, or the efficiency of competitive equilibria.

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[1]See, for instance, Barro (1976), Feldstein (1976) or Blanchard (1985).

[2]E.g. Obstfeld and Rogoff (1983).

[3]See Aiyagari (1985) for a simple proof.

[4]See Tirole (1982).

To clarify these issues, I construct a simple model without a representative consumer which, *in spite of the infinite horizon of every agent alive*, yields results traditionally associated with overlapping generation (OLG) economies: debt is non-neutral, asset bubbles can exist, and competitive equilibria may be dynamically inefficient. The model abandons the finite lifetime assumption of OLG models (which, I shall argue, is inessential), but maintains the crucial hypothesis that new cohorts enter the economy over time; it is, therefore, a model of 'overlapping families of infinitely-lived agents'. This framework is shown below as resulting from the existence of operative intergenerational linkages between *some but not all* agents, partial linkages which are sufficient to endow any agent alive at any date with an effectively infinite economic horizon.

The paper is organized as follows. Section 2 lays out the basic demographic structure of the economy and describes the individual and aggregate optimum consumption programs. Section 3, by characterizing a pure exchange equilibrium with three different asset menus, establishes that this model with overlapping and infinitely-lived families possesses many of the features of the standard life-cycle model. Section 4 discusses the importance of infinite lives for the theory of asset bubbles and the Ricardian debt neutrality proposition. The conclusion summarizes the results.

2. The basic framework

2.1. Demography

The economy consists of many infinitely-lived dynasties. In an interval of time of length $\mathrm{d}t$, $\dot{N}(t) \equiv \mathrm{d}N(t) = nN(t)\,\mathrm{d}t$, $n \geqq 0$, *new* and identical infinitely-lived families appear in the economy, so that the total number of families alive at time t is $N(t) = N(0)\mathrm{e}^{nt}$.

A *new* family is defined as one which is not linked through operative intergenerational transfers to pre-existing dynasties. A few examples illustrate this definition. Consider, for instance, a primogeniture economy in which a parent only loves his first born heir, enough to leave him a bequest.[5] Assume that all parents have children, and that children do not love their parents. Each child, *whether first-born or not*, is, in this economy, linked through operative bequests to the never-ending chain of his first-born descendants, and is thus part of an infinitely-lived family. Children who are not first-born, however, do not belong to any pre-existing dynasty, since they were not loved by their parents: they initiate the dynasty to which they belong. The rate at which new dynasties enter the economy is a reflection, in such an environment, of the proportion of children who are not loved, or not

[5]See Weil (1987) for conditions for operative bequests.

loved enough, by their parents. Envisage, alternatively, an economy in which a Salic law bars women from inheriting from their parents. But for the complications which arise when reproduction by parthenogenesis is abandoned and gender differences are introduced,[6] this economy is similar to the one with primogeniture. Imagine, finally, a world in which all parents love and are economically linked to all of their children but in which immigrants are completely cut off from their relatives in their home country. Newly arrived immigrants initiate, in such a world, new infinitely-lived dynasties. More generally, the speed of arrival of new cohorts, n is a measure of the economic disconnectedness of the population, and captures, beyond strict socio-demographic interpretations, its heterogeneity.

At a more abstract level, this model can be viewed as a standard overlapping generations economy without intergenerational altruism in which the lifespan of every agent would have been stretched to infinity.[7] Agents (dynasties) are thus born, but never die. Alternatively, this framework can be thought of as an extreme version of Blanchard's (1985) model in which the instantaneous death probability, *but not the birth rate*, would have been set to zero.[8]

Whichever interpretation one prefers, the economy is characterized by Shell's (1971) 'double infinity' of traders and dated commodities, so that one should not be surprised, from a general equilibrium standpoint, to find below that competitive equilibria might be dynamically inefficient – and this in spite of the infinite horizon of every agent alive.

Under the additional, and inconsequent, simplification that there is no intra-family growth,[9] the parameter n measures both the speed of arrival of new cohorts and the rate of growth of the population.

2.2. Consumers

Adopting Blanchard's (1985) notation and closely following the style of his derivations to highlight the striking formal parallelism between his and this

[6]See, for instance, Nerlove, Razin and Sadka (1984), and Bernheim and Bagwell (1988).

[7]In contrast with Aiyagari (1985), who has independently worked on related issues, I assume that horizons are literally infinite, and not merely arbitrarily large.

[8]Blanchard's (1985) derivations assume that the birth and death rates are equal (so that population is constant), making it impossible to disentangle, in his model, the effects of the birth of new, unrelated cohorts from those of finite horizons. To highlight the importance of 'disconnectedness', I simply eliminate death from my model, and instead concentrate on birth. On the basis of Blanchard's (1985) work and of an earlier version of this article [Weil (1985)], Buiter (1988) completes the analysis by building a model à la Yaari (1965) in which the birth and death rates are distinct.

[9]Intra-cohort growth is straightforward to introduce. It suffices to reinterpret all the per capita magnitudes which I define below as being, in fact, measured per family. Intra-cohort growth does not introduce any additional disconnectedness into the economy, and thus does not affect any of the results but for a scale factor.

model, I let $c(s,t)$, $y(t)$,[10] and $w(s,t)$ denote, respectively, the consumption, noninterest disposable income, and nonhuman wealth at time t of an agent born at time $s \leqq t$, $s \geqq 0$. Letting $r(t)$ denote the real rate of interest at t, and assuming that agents' tastes can be represented by a logarithmic[11] utility function with positive instantaneous subjective discount rate δ,[12] an agent representative of the cohort born at time s chooses $c(s,t)$ and $w(s,t)$, $t \geqq s$, to maximize

$$U(s) = \int_s^\infty e^{-\delta(t-s)} \ln c(s,t)\,dt \tag{1}$$

subject to the instantaneous budget constraint:

$$dw(s,t)/dt = r(t)w(s,t) + y(t) - c(s,t), \tag{2}$$

and under the restrictions that

$$\begin{cases} c(s,t) \geqq 0, & \forall t \geqq s \geqq 0, \\ w(s,s) = 0, & \forall s > 0, \end{cases} \tag{3}$$

$$\lim_{t\to\infty} w(s,t)e^{-R(s,t)} \geqq 0, \tag{4}$$

where $R(s,t)$ denotes the compounded interest rate between s and t, i.e.

$$R(s,t) \equiv \int_s^t r(x)\,dx. \tag{5}$$

Eq. (3) states that, except possibly for the cohort born at the origin of time which might find some positive nonhuman wealth lying in the Garden of Eden, nonhuman wealth at birth is zero. This constraint reflects the definition of a new cohort. Condition (4) prevents *private* agents from rolling

[10]Noninterest income is age independent, as there is no meaningful sense in which dynasties may retire, and because all lump-sum taxes and transfers are assumed to be age independent. As Aiyagari (1985) has shown in a two-period overlapping generations model, this specification of the tax distribution, although fairly natural, is not innocuous.

[11]The assumption of logarithmic utility can easily be relaxed to allow for HARA preferences. Such a relaxation does not affect the results qualitatively, and offers no additional insight into the model. Other utility functions yield consumption functions which are nonlinear in wealth and thus cannot be aggregated across the whole population.

[12]This subjective discount rate reflects both pure time preference and intrapersonal discounting: consumption today is less enjoyable than consumption tomorrow, and my own consumption pleases me more than my heirs'. There is no need to separate the two concepts at the level of abstraction of this paper.

their debt over forever. The solution to this maximization problem is fully characterized by (2), (3) and

$$c(s,t)=\delta[w(s,t)+h(t)], \tag{6}$$

$$\mathrm{d}c(s,t))/\mathrm{d}t=[r(t)-\delta]c(s,t), \tag{7}$$

where

$$h(t)\equiv\int_t^\infty y(v)\mathrm{e}^{-R(s,v)}\,\mathrm{d}v \tag{8}$$

denotes human wealth at t, which is identical for all agents alive at t as noninterest income is, by assumption, age independent. The consumption function (6) embodies the fact that an optimum program satisfies (4), and thus the lifetime budget constraint which I have not written, with equality. The propensity to consume out of total wealth, δ, is age independent, because all agents alive have the same, infinite horizon.[13] It is constant because of the logarithmic utility specification. From (7), individual consumption grows at the rate $r-\delta$, a familiar result.

For any variable $x(s,t)$ pertaining to an individual agent, define the corresponding per capita aggregate magnitude $X(t)$ as

$$X(t)=\left[N(0)x(0,t)+\int_0^t x(s,t)\,\mathrm{d}N(s)\right]\Big/N(t). \tag{9}$$

Using this definition, it follows from (6) that per capita aggregate consumption is a constant fraction δ of total per capita wealth,

$$C=\delta[W+H], \tag{10}$$

and, from (2) and (8), that per capita aggregate nonhuman and human wealth evolve according to

$$\dot{W}=(r-n)W+Y-C, \tag{11}$$

$$\dot{H}=rH-Y, \tag{12}$$

with time arguments omitted when no ambiguity results, and $Y(t)=y(t)$ and $H(t)=h(t)$ under our assumptions. Per capita aggregate nonhuman and human wealth do not accumulate at the same rate, a reflection of the fact

[13]This feature of the model makes aggregation and the derivation of closed form solutions possble, since, as in Blanchard (1985), all agents alive have the same (here infinite) remaining lifetime.

that while newly born cohorts have by definition no nonhuman wealth, their human wealth is identical to that of pre-existing dynasties. From the three previous equations, the law of motion of per capita aggregate consumption is given by

$$\dot{C}=(r-\delta)C-n\delta W, \tag{13}$$

which is the fundamental equation of this paper. If $n=0$, the economy consists only of the original infinitely-lived family, and (14) reduces to the standard condition $\dot{C}/C=r-\delta$ of the representative family model with logarithmic utility. When n is positive, the aggregate and individual trajectories of consumption differ, because the newly born have, by definition, zero nonhuman wealth.

2.3. Government

Let $T(t)$ and $B(t)$ denote, respectively, lump-sum taxes (transfers if negative) and the real value of the stock of a public liability which I will descibe below, all measured in aggregate per capita terms. Assuming that government spending is zero, the government instantaneous budget constraint is then given by

$$T+\dot{B}=(r-n)B. \tag{14}$$

Note that, because of the assumption made above that lump-sum taxes are age independent, $T(t)$ also represents the lump-sum taxes paid by any individual alive at time t.

3. Exchange economy

In this section I characterize the competitive equilibria of an economy in which output is nonproduced and nonstorable.[14] Each agent alive is assumed to receive at any given instant of time an endowment $e>0$ of the consumption good, so that per capita aggregate disposable income is simply

$$Y=e-T. \tag{15}$$

Equilibrium in the consumption good market then requires that per capita aggregate consumption be constant at its market-clearing level, e:

$$C=e, \tag{16}$$

[14]The equilibrium with neoclassical production is characterized in the appendix.

so that, from (11), (13) and (15):

$$\dot{C}=(r-\delta)e-n\delta W=0, \tag{17}$$

$$\dot{W}=(r-n)W-T. \tag{18}$$

In equilibrium, individual consumption must be non-negative at every instant. From (3), (6) and (7) it is immediate to see that this will be satisfied, when $n>0$, if and only if human wealth $h(s)$, and hence consumption of the newborn is non-negative for all $s>0$. But $h(s)=H(s)\geqq 0$ is equivalent, from (10), to $W\leqq C/\delta$, which, from (16) and (17), simply requires that

$$r(t)\leqq n+\delta, \quad \forall t>0.^{15} \tag{19}$$

To close the model, I consider three different asset menus. In the first one, a 'nonmonetary'. economy, neither government liabilities nor intrinsically useless assets (bubbles) circulate. In the second one, a 'monetary' economy, an unbacked and intrinsically useless fiat currency provides a nonphysical store of value. In the third one, government debt circulates.

3.1. Nonmonetary economy

In the absence of government liabilities ($B=0$) or of a 'bubbly' asset, all nonhuman wealth must be held in the form of consumption loans, since output is perishable. Clearing of the private credit market therefore implies that aggregate nonhuman wealth is zero in equilibrium:

$$W(t)=0, \quad \forall t\geqq 0. \tag{20}$$

Note that this condition imposes that $W(0)=w(0,0)=0$, as required when the only store of value is constituted by consumption loans.

From (18), taxes are zero in this equilibrium without government activity. More interestingly, the equilibrium interest rate is simply, from (18), equal to the rate of time preference:

$$r(t)=\delta, \quad \forall t\geqq 0. \tag{21}$$

From (7), individual consumption is constant over time, so that, given the flat lifetime noninterest income profile, the equilibrium is intergenerationally autarkic. Heuristically, this can be understood by noting that this economy

[15]A similar inequality appears in Blanchard (1985). Note, from (17), that the steady-state interest rate is not necessarily bounded from below by δ if $W<0$, i.e. if there is an outside claim on the private sector in steady state (e.g. if the government holds claims on the public).

starts at its 'biological' origin. The initial family born at $t=0$ cannot trade, at $t=0$, with the agents to be born even an infinitesimal instant later; it therefore consumes its endowment, and does not accumulate any nonhuman wealth. One instant of time later, the initial family and the newly born dynasty can in principle trade with each other. However, they do not, in practice do so, as they are identical; the have the same zero nonhuman wealth, the same human wealth, and the same infinite horizon. This reasoning implies, by induction, that dynasties never trade with each other in this equilibrium. The subjective discount rate, δ, therefore stands, in this economy with logarithmic preferences and flat noninterest income profiles, for the intergenerationally autarkic interest rate.

A very important implication of eq. (21) is that the autarkic interest rate, δ, bears no necessary relationship to the rate of population growth, n – unless, of course, n is zero. This competitive equilibrium can therefore be dynamically inefficient, if $\delta < n$, despite the infinite horizon of every agent alive. Although this result is not surprising from a general equilibrium standpoint [see Shell (1971)], it shows, from a methodological perspective, that it is easy to endow the neoclassical model of infinitely-lived agents with features typical of standard overlapping generation models, and that infinite horizons do not imply dynamic efficiency when there is no representative agent.

3.2. Monetary economy

The introduction of an unbacked and intrinsically useless asset, which I call money for convenience, further illustrates that this model with infinite lifespans has properties which are very similar to those of the life-cycle model – provided that n, the degree of disconnectedness, be positive.

Suppose that intrinsically useless pieces of paper, 'money', are given by the government to the first cohort at the origin of time, so that the nonhuman wealth at $t=0$ of this original family is positive if these pieces of paper are valued. Assume, in addition, that the government increases the nominal money supply at the constant rate σ, and redistributes seigniorage equally among all agents alive at any given point in time. The per capita helicopter drop of money which every agent alive at t receives has real value:

$$T(t) = -\sigma B(t), \tag{22}$$

where $B(t)$ denotes, in accordance with the definition in subsection 2.3, per capita aggregate real balances.

In equilibrium, all nonhuman wealth must be held, in the aggregate, in the form of money (since the inside credit market must clear):

$$W = B. \tag{23}$$

From (17), (18), (22) and (23) it must therefore be the case that

$$\dot{C}=(r-\delta)e-n\delta B=0, \tag{24}$$

$$\dot{B}=(r-n+\sigma)B. \tag{25}$$

These two equations characterize, for a given σ, the equilibrium perfect foresight dynamics of the real interest rate and of real balances.

The dynamic system, (24) and (25), has two steady states. One is, of course, the nonmonetary equilibrium in which money is not valued ($B=0$) and the interest rate is equal to the rate of time preference ($r=\delta$). The other steady state is one in which B is nonzero, with the real rate of return on consumption loans being equal to the real rate of return on money, i.e.

$$r=n-\sigma\equiv r^*. \tag{26}$$

A necessary condition[16] for this steady state to be a valid monetary equilibrium is that the associated real balances $B^*=e(r^*-\delta)/n\delta$ be positive. This requires that $r^*>\delta$, or, from (26), that

$$\sigma<n-\delta. \tag{27}$$

This inequality is simply, in this continuous time model with logarithmic utility, the equivalent of Wallace's (1980) condition on the maximum rate of growth of the nominal money supply consistent with the existence of a monetary equilibrium. One implication of (27) is that, as in standard overlapping generations models, an equilibrium in which the nominal money supply is nondecreasing can only exist if $\delta<n$, i.e. if the nonmonetary economy is dynamically inefficient. Another implication is that $r^*>\delta$ in equilibrium, to that individual consumption rises over time.[17]

The analysis of dynamic paths is illustrated in figs. 1 and 2 for the case of a constant nominal money supply ($\sigma=0$). If, on the one hand, the intergenerationally autarkic equilibrium is dynamically inefficient ($\delta<n$), then there exists a continuum of perfect foresight equilibria indexed by $B(0)$, the initial real balances. It must be the case that $B(0)\in[0,B^*]$, since otherwise either real balances or the consumption of the newly born become negative in finite time; all monetary equilibria starting with $B(0)$ (positive but strictly below B^*

[16]An additional condition, which is necessary and sufficient in conjunction with (26), follows, when $n>0$, from (19): $r^*\leqq n+\delta$. This requires that $\sigma\geqq-\delta$, a condition which is henceforth assumed to be satisfied.

[17]This is consistent with the existence of an aggregate (per capita) steady state because the high consumption rate of the relatively old dynasties is balanced by the low consumption of the relatively young families.

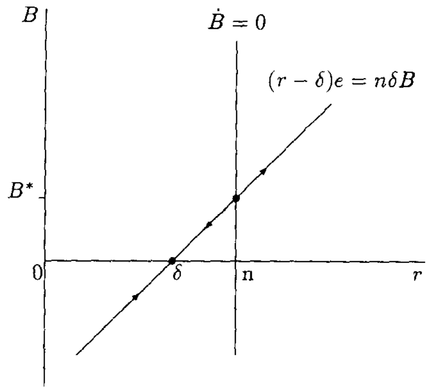

Fig. 1. Determination of equilibrium dynamics ($\delta < n$).

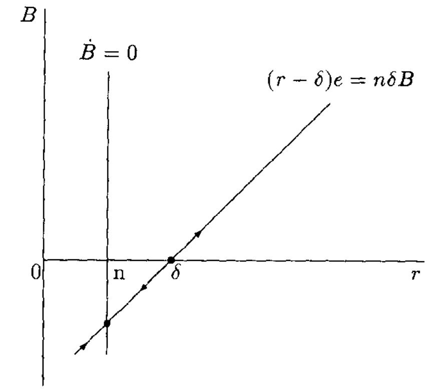

Fig. 2. Determination of equilibrium dynamics ($\delta > n$).

converge to the nonmonetary steady state. If, on the other hand, the nonmonetary economy is dynamically efficient ($\delta \geqq n$), no monetary equilibrium, whether stationary or nonstationary, exists; money cannot be valued, and the only equilibrium is the nonmonetary steady state with $r = \delta$. These results are similar to those which can be derived from a standard two-period overlapping generations model [see, for instance, Tirole (1985].

The introduction of 'money', when the conditions for its being valued are satisfied, makes interdynastic trade possible for the following reason. At the origin of time, the original dynasty still has nobody to trade with. But it is, by assumption, endowed with the initial money stock, and thus has at time $0 + \mathrm{d}t$ a larger nonhuman wealth than the family born at that instant. This difference, and this difference alone, forms the basis for interdynastic trade.

3.3. Public debt

The introduction of a different government liability, public debt, only requires relabeling the useless asset called 'money' in the preceding section. In order to provide some additional insight, consider, however, a financing rule slightly different from the one analyzed above: fiscal authorities aim at maintaining the per capita stock of public debt constant at some level $\bar{B}>0$, which, from (14), involves levying lump-sum taxes $T=(r-n)\bar{B}\equiv\bar{T}$. In equilibrium, in the absence of 'money', all aggregate human wealth must be held in the form of government bonds, so that $W=\bar{B}$. Therefore, from (17), the equilibrium interest rate is given by

$$r(t)=\delta+n\delta\bar{B}/e\equiv\bar{r}>\delta, \quad \forall t\geqq 0. \tag{28}$$

If n is positive, the equilibrium interest rate, and hence the equilibrium allocation of the aggregate endowment among dynasties, is not independent of the government financing decision as embodied in $\bar{B}$, and $\bar{r}$ increases when $\bar{B}$ rises. To understand why, it suffices to realize that government bonds are net wealth in this economy if $n>0$, since the excess of the value of the public debt held by agents alive today over the present discounted value of the tax liabilities to be incurred in the future *by those very same agents*, is simply

$$\bar{B}-\bar{T}/\bar{r}=n\bar{B}/\bar{r}, \tag{29}$$

where is strictly positive if both n and $\bar{B}$ are positive. What happens is that part of the future taxes associated with the public debt will be paid by dynasties which are not yet born and whose consumption the dynasties alive today do not value. As a consequence, a bond financed postponement of taxes makes all dynasties alive today better off (and thus induces them to consume more), not because they might not be alive when future taxes are levied (they will, as they live forever), but because the future tax base will include new agents to whom they are not economically connected.[18] The real interest rate must hence rise to maintain aggregate consumption at its market-clearing level, as implied by (28). Infinite lifetimes are therefore not inconsistent with the violation of the Ricardian debt neutrality proposition.

Another potential source of violation of Ricardian neutrality, which is robust to modifications of the interdynastic tax distribution, is the fact that, as above, nothing guarantees that $\bar{r}\geqq n$ when new cohorts enter the economy ($n>0$). As a consequence, it is possible for the government to transform its debt into a Ponzi scheme when $\bar{r}<n$, which would result in an obviously

[18]This argument would not, of course, be valid if the beneficiaries of the tax cut were to pay all of the additional taxes associated with the increased debt.

non-neutral Pareto improvement. Infinite lifetimes thus do not necessarily imply the existence of an intertemporal budget constraint on the government.

4. Discussion

I now analyze the further implications of the foregoing results for the role of finite lives in macroeconomics, in relation to the Ricardian debt neutrality proposition and the theory of asset bubbles.

4.1. Infinite lives and Ricardian neutrality

The model analyzed in this paper illustrates that there is no direct link between the length of dynastic horizons and the satisfaction or violation of the Ricardian debt neutrality proposition. There is a sense, of course, in which this result is known, in that it is well understood, within the context of the standard overlapping generation model, that it is the economic identity of future taxpayers which determines whether Ricardian neutrality holds or not [see, for instance, Aiyagari (1985)]. Yet, substantial confusion has arisen in the literature regarding the links between operative intergenerational transfers, infinite dynastic horizons, and the Ricardian debt neutrality proposition. It is in particular widely argued that operative intergenerational transfers between all generations, *because* they imply infinite horizons, lead to Ricardian neutrality (cf. the debate between Feldstein (1976) and Barro (1976)]. It is also suggested that finite lifespans lead to the violation of the Ricardian proposition [see Blanchard (1985)]. I now turn, to evaluate and qualify them, to those two lines of reasoning.

Both partial intergenerational linkages, such as the ones on which this model is based, and complete interdynastic links result in every dynasty having an infinite horizon. Yet Ricardian neutrality in general holds (with lump-sum taxes) in the latter but not in the former framework. This shows that it is not possible to draw conclusions on the validity of the Ricardian proposition from the fact that all agents alive have infinite horizons. The crucial distinction between the two frameworks is the degree to which population is economically connected, in the sense defined above.

In an analogous, but somewhat subtler, way, finite horizons do not necessarily imply the violation of Ricardian neutrality.[19] Consider for instance an economy, polar to the one studied in this model, in which nobody is ever born but in which agents die over time, and with compulsory annuitization of wealth as in Blanchard (1985). A postponement in taxes, balanced in present value terms, has two opposite effects on the human wealth of those alive today. On the one hand, an agent alive today might be

[19]This paragraph draws on Buiter's (1988) synthesis of Blanchard's (1985) model of an earlier [Weil (1985)] version of the model developed here.

dead when increased taxes are levied in the future. On the other hand, he will have to pay higher taxes if he survives, since the number of taxpayers will have decreased. The sign of the total effect can easily be shown to be ambiguous in the sense that it depends on the assumed distribution of death probabilities. Generically the total effect is nonzero and thus Ricardian neutrality does not hold. There is, however, a very interesting special case in which the instantaneous death probability, conditional on having survived, is constant: i.e. the exponential distribution of death dates used by Blanchard (1985). In that case, and more generally when all surviving agents have the same horizon (whether stochastic or deterministic), it is easy to show[20] that the two effects cancel each other out. As a consequence, Ricardian neutrality holds in spite of finite horizons. An implication of this surprising result is that finite horizons are not sufficient to generate the Ricardian non-neutralities in Blanchard's (1985) model; the arrival of new disconnected cohorts is necessary for that purpose. This should not be construed, however, as implying that finite horizons do not matter in Blanchard's (1985) model; they do, but only by *reinforcing* the non-neutralities introduced by the arrival of new disconnected cohorts. In general, therefore, the effects of disconnectedness and finite horizons compound each other.

4.2. *Infinite lives and asset bubbles*

Section 3 established that it is possible, subject to some familiar existence conditions, for an intrinsically useless and unbacked asset, 'money', or more generally a bubble to be valued in equilibrium in spite of the infinite lifespan of every agent alive – provided that new dynasties enter the economy over time.

This demonstrates that the feature of the overlapping generations model, which is crucial for the existence of asset bubbles, is not finite horizons (the fact that people die), as is sometimes argued,[21] but rather the constant arrival of new cohorts into the economy (the fact that disconnected agents are born), as is implicit from Tirole's (1982) article. In other words, the transversality conditions associated with the optimal consumption program of infinitely-lived agents do not in general rule out the existence of asset bubbles, as documented in this model. Complete economic connection, in the form of operative transfers, between *all* generations present and future (which

[20]The proof is a direct implication of Buiter's (1988) model: if no new dynasties enter the economy and wealth is annuitized, but cohorts die exponentially (so that they have the same life expectancy), then Ricardian neutrality holds. Abel (1989) elaborates this point and shows, in a different setup, that finite horizons and debt neutrality are compatible only if annuities markets are open; the crucial feature of his model is that all agents alive at any given date have the same remaining lifetime.

[21]See, for instance, Obstfeld and Rogoff (1983).

implies, but is not equivalent to, infinite dynastic lifespan) would however prevent the existence of bubbles.

5. Conclusion

This model has established that finite horizons are not necessary to generate most of results usually associated with the overlapping generations model. The crucial ingredient of the model is the continuous arrival into the economy of new, infinitely-lived dynasties which are, by definition, not part of pre-existing families in the sense that they are not linked to older cohorts through operative interdynastic transfers.

From a methodological perspective, this model provides a convenient, although parametric, hybrid of the overlapping generations model and infinite horizon representative agent framework. It combines the life-cycle features of the former, while retaining the analytical simplicity entailed, in the former, by infinite horizons.

From a theoretical standpoint it has been shown that infinite horizons are consistent with dynamic inefficiency, the valuation of intrinsically useless and unbacked assets, the existence of asset bubbles, and the violation of Ricardian neutrality. All that is needed to allow for these phenomena is the existence of some degree of economic disconnectedness between dynasties (which implies the absence of a representative agent).

While the importance of these phenomena is clearly dependent, for practical purposes, on the conjectured magnitude of 'disconnectedness', it seems difficult not to accept, at least as the working hypothesis upon which much of classical economic theory is built, that some degree of selfishness, and thus disconnectedness, characterizes the objective functions of economic agents.

Appendix

In this appendix I briefly characterize the properties of the intergenerationally autarkic equilibrium (without nonphysical stores of value) in a one-good environment with neoclassical production.

Let K denote, to be consistent with the notation in the text, the aggregate capital–labor ratio, and $f(\cdot)$ be the constant returns to scale production function in intensive form, which satisfies the usual curvature assumptions. Competitive profit maximization by firms ensures that the interest (wage) rate is equalized to the marginal productivity of capital (labor), so that

$$r = f'(K), \tag{A.1}$$

$$\omega = f(K) - Kf'(K), \tag{A.2}$$

where time arguments have been deleted for notational convenience. Agents' noninterest income simply consists, in the absence of taxes or transfers, of the wage the receive, assuming that one unit of labor is inelastically supplied by each dynasty. Therefore

$$Y = y = \omega. \tag{A.3}$$

In equilibrium, all nonhuman wealth is held in the form of real capital, so that it must be the case that

$$W = K. \tag{A.4}$$

Substituting (A.1) – (A.4) into (11) and (13) in the text, the equilibrium laws of motion of per capita aggregate consumption and of the capital–labor ratio are simply:

$$\dot{C} = [f'(K) - \delta]C - n\delta K, \tag{A.5}$$

$$\dot{K} = f(K) - nK - C, \tag{A.6}$$

with the initial capital stock $K(0)$, i.e. $w(0,0)$, given. Notice that when $n=0$ the dynamics are those of the standard representative agent model with logarithmic utility, with the interest rate tied, in the long run, by the modified golden rule, δ.

In general, when $n>0$, dynamics can be represented in a phase diagram in (K, C) space. The $\dot{C}=0$ locus is upward sloping, with a vertical asymptote at K^*, where $f'(K^*)=\delta$. The $\dot{K}=0$ locus reaches a maximum at the golden rule capital stock $\hat{K}$, with $f'(\hat{K})=n$. Assume that the intersection of these two loci exists and is unique (which involves imposing restrictions, which I do not study, on the production function).[22] It is then straightforward to show that the steady state $(\tilde{K}, \tilde{C})$ is saddlepoint stable, and that the saddlepath trajectory leading to this steady state is the unique equilibrium trajectory.

The following are a few of the steady-state properties of the model. The long-run interest rate is always larger than δ, but smaller than $n+\delta$ (since $\tilde{H}=\tilde{\omega}/\tilde{r}$ is positive by construction). More interestingly, there is no a priori presumption as to whether $\tilde{r}$ is below or above n (except if $n=0$, which implies that $\tilde{r}=\delta>0$, or if $\delta>n$, which suffices to entail that $\tilde{r}>n$ since $\tilde{r}>\delta$).

[22]The equilibrium steady-state capital stock $\tilde{K}$ must solve $(\tilde{r}-\delta)\tilde{C}= n\delta\tilde{K}$, $\tilde{C}=f(\tilde{K})-n\tilde{K}$, $\tilde{r}=f'(\tilde{K})$. Now, in steady state, $H=\tilde{\omega}/\tilde{r}$, so that, from the consumption function (10), $\tilde{C}=\tilde{K}+\tilde{\omega}/\tilde{r}$. Therefore, after some manipulations, one finds that $\tilde{K}$ must solve $n\delta/[(n+\delta-\tilde{r})\tilde{r}]=[f(\tilde{K})-n\tilde{K}]/[f(\tilde{K})-\tilde{r}\tilde{K}]$. Note the similarity of this equation with (11) in Cass and Yaari (1967): the exponential terms in their equation drop out when lifespans are increased from one unit of time to infinity.

This feature, characteristic of Diamond's (1965) model, is thus consistent with infinite horizons. Finally, when disconnectedness (n) increases, the $\dot{C}=0$ locus shifts upward, so that the steady-state capital stock $\tilde{K}$ decreases. Unlike in the exchange economy, the interdynastically autarkic steady-state interest rate is thus not dependent of n.

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