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PERMANENT BUDGET DEFICITS AND INFLATION*

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The issue of whether permanent primary budget deficits have to be monetized is re-examined in a simple monetary model, hybrid of the Sidrauski and overlapping-generations frameworks, in which intergenerational effects are generated by the arrival of new infinitely-lived cohorts. The presence of intergenerational effects enlarges, for a given fiscal policy, the set of admissible monetary policies and weakens the need to monetize the deficit. Comparisons between the growth and interest rates are in general insufficient to predict whether increased deficit must be monetized, because of the existence of both bond and money seigniorage Laffer curves.

1. Introduction

One of the motivations of the Sargent and Wallace (1981) analysis of permanent budget deficits was to explore the nature of the restrictions imposed by general equilibrium upon monetary authorities in a policy regime characterized by the dominance and independence of fiscal authorities. An 'unpleasant' implication of the Sargent–Wallace overlapping-generations model, namely that additional permanent primary deficits must eventually be monetized, was criticized by Darby (1984) on the grounds that it could be reversed in a model in which the real interest rate was below the rate of growth of population. Miller and Sargent (1984) retorted that, in a model in which the assumption of an exogenously fixed interest rate were abandoned (that assumption was common to the Sargent–Wallace and Darby analyses), simple comparisons between real interest rates and growth rates would in general prove insufficient to settle the issue, because of the existence of a bond revenue Laffer curve. Their model was not, however, firmly grounded in general equilibrium and appealed to ad hoc demand functions for bonds and money.

The Sargent–Wallace overlapping-generations results were, at the same time, re-examined and, by and large, confirmed by McCallum (1984), Liviatan (1984) and Drazen (1984) within the context of a representative, infinitely-lived dynasty model à la Sidrauski (1967), with the demand for money deriving

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from the insertion of real balances in the utility function. These analyses, more satisfactorily derived from first principles and cast within an Arrow–Debreu framework, eliminated, however, the very source of Darby's counterargument to Sargent and Wallace, namely the potential dynamic inefficiency of the economy.

In this article, I present a synthesis of these two strands of literature by using a simple monetary model, hybrid of the Sidrauski (1967) framework with money in the utility function and of the overlapping-generations model, in which new and infinitely-lived dynasties continuously enter the economy. The reasons for using this model are twofold. On the one hand, it provides a convenient measure, the rate of arrival of new cohorts, of the strength of the 'intergenerational effects' which are crucial to Darby's conclusion, and in particular embodies the Sidrauski model, and thus McCallum's analysis, as a special case. On the other hand, it is at the same time a model in which asset demand functions are derived from first principles, and interest rate and prices endogenously determined. It is thus particularly well suited to a re-examination of the issues raised by Sargent and Wallace.

Three main conclusions will emerge from this study. Firstly, that the set of admissible monetary policies is likely to be much wider in the presence of intergenerational effects than it is in the representative family framework. Secondly, that, as Miller and Sargent suggested, simple comparisons between the real interest rate and the growth rate are not sufficient to predict in general whether additional permanent deficits must be monetized, because of the presence of Laffer curves for *both* bond and money seigniorage. Finally, that the purchase by future but yet unborn cohorts of part of the public debt issues associated with the financing of the deficit makes it possible, in most cases, for monetary authorities to monetize a smaller fraction of any given permanent deficit.

This article is organized as follows. The next section lays out the model, examines the problems faced by consumers and government, and presents equilibrium conditions. Section 3 characterizes admissible monetary policies, both in the absence and in the presence of intergenerational effects. Section 4 concludes this study.

2. The model

The basic framework is a simplified monetary version of the model of overlapping and infinitely-lived dynasties which I have developed elsewhere.[1] It is a hybrid of the Sidrauski (1967) model with money in the utility function and of the life-cycle model.

[1] See Weil (1985, 1986).

The economy consists of many infinitely-lived dynasties. At the origin of time, there are $N(0)$ identical families alive. During an interval of time of length $\mathrm{d}t$, $\mathrm{d}N(t) = nN(t)\,\mathrm{d}t$, $n \geq 0$, new and identical infinitely-lived families appear in the economy, so that total population alive at time $t \geq 0$ is $N(t) = N(0)\mathrm{e}^{nt}$. New cohorts are, by definition, newly created dynasties which are not linked to pre-existing ones through operative bequests. In this model with no intra-cohort growth,[2] n both measures the rate of growth of population and parametrizes the degree of 'disconnectedness' of the economy, that is, the extent to which the model departs from the infinitely-lived, representative agent paradigm. Thus, when $n = 0$, the economy reduces to the $N(0)$ original, identical and infinitely-lived families while, when n is strictly positive, new infinitely-lived dynasties enter the economy over time. This framework is most easily thought of as a standard overlapping-generations model in which the lifespan of agents would have been stretched to infinity. A more structural demographic approach would interpret each new dynasty as being initiated by a child not linked through bequests to his/her parents, and as containing an infinite chain of descendants linked through bequests. In a primogeniture economy, for instance, new dynasties would be started by children who are not first-born, and would include the infinite chain of their first-born heirs. The model thus stresses the importance of *economic* disconnectedness, as captured by n, in generating life-cycle type results, by contrast with Blanchard's (1985) analysis which, seemingly,[3] highlights the role of finite horizons.

The economy is monetary, with money yielding a flow of utility services proportional to the stock of real balances. This assumption is embodied, for convenience, by the insertion of real balances in the consumers' utility function, as in Sidrauski (1967).

2.1. Consumers

A family representative of the cohort born at times $s \geq 0$ maximizes

$$\int_{s}^{+\infty} \mathrm{e}^{-\delta(t-s)} u[c(s,t), m(s,t)]\,\mathrm{d}t, \tag{1a}$$

with the utility function being given by

$$u(c,m) = (1-\mu)\log c + \mu \log m, \qquad 0 \leq \mu < 1, \tag{1b}$$

[2] Intra-cohort growth, which does not introduce additional heterogeneity into the population and does not qualitatively affect the results of this article, is ignored for simplicity.

[3] In Blanchard's (1985) paper, the rate of arrival of new cohorts is equal, by construction, to the probability of death, so that one cannot disentangle the effects of disconnectedness (arrival of *new* cohorts) from those of finite horizons (positive death probability). Weil (1985) shows that disconnectedness, and not finite lifetimes, is crucial in generating life-cycle results in the Yaari–Blanchard framework. This conclusion is confirmed by Buiter (1986), who dissociates rates of arrival of new cohorts and of death in the Blanchard (1985) model, and by Abel (1987).

subject to

$$a(s,t) = w(s,t) + m(s,t), \tag{2a}$$

$$\mathrm{d}a(s,t)/\mathrm{d}t = r(t)w(s,t) - \Pi(t)m(s,t) + y(t) - c(s,t), \tag{2b}$$

$$\lim_{t\to\infty} \exp\left\{-\int_s^t r(x)\,\mathrm{d}x\right\} a(s,t) \geq 0, \tag{2c}$$

$$c(s,t), m(s,t) \geq 0, \tag{2d}$$

$$a(0,0) = a_0 > 0, \tag{2e}$$

$$a(s,s) = 0, \qquad s > 0, \tag{2f}$$

where $c(s,t)$, $m(s,t)$, $w(s,t)$, $a(s,t)$ and $y(t)$ represent, respectively, consumption, real money balances, non-human wealth held in non-monetary assets, non-human wealth, and non-interest income[4] at time $t \geq s$ of a family born at time s. $r(t)$, $\Pi(t)$ and $\delta > 0$ are, respectively, the instantaneous real rate of interest, inflation rate, and subjective rate of time preference. The parameter μ measures the contribution of real balances to utility. Constraint (2c) prevents each family from rolling its debt over forever. The initial conditions $a(s,s) = 0$, $s > 0$, reflect definition of a new cohort as being a family not linked to previous dynasties through operative bequests. Only the original family may find some non-human wealth lying, at birth, in the Garden of Eden, namely $a(0,0) = a_0$.

Using the definition, $i(t) \equiv r(t) + \Pi(t)$, of the nominal rate of interest to rewrite the instantaneous budget constraint as

$$\mathrm{d}a(s,t)/\mathrm{d}t = r(t)a(s,t) + y(t) - [c(s,t) + i(t)m(s,t)], \tag{3a}$$

where $c + im$ represents 'full' consumption, it is straightforward to show that the optimum individual program is fully described by the initial conditions (2e) and (2f) and

$$c(s,t) = \delta(1-\mu)[a(s,t) + h(t)], \tag{3b}$$

$$\mathrm{d}c(s,t)/\mathrm{d}t = r(t) - \delta, \tag{3c}$$

$$m(s,t) = [\mu/(1-\mu)][c(s,t)/i(t)], \tag{3d}$$

[4]Non-interest income is age-independent, to reflect the fact that each dynasty should be considered as a chain of finitely-lived agents linked by operative bequests. There is, in fact, no meaningful sense in which 'old' families can be distinguished, from the point of view of their non-interest income, from 'young' dynasties. In particular, dynasties do not 'retire'.

where

$$h(t) \equiv \int_t^{\infty} y(v) \exp\left\{-\int_t^{v} r(x)\,\mathrm{d}x\right\} \mathrm{d}v \tag{3e}$$

is the human wealth of a family alive at time t (all families alive have the same human wealth because they have identical, infinite horizons and receive age-independent non-interest income). From (3b), consumption is a constant fraction $\delta(1-\mu)$ of total (non-human plus human) wealth; equivalently, using (3d), 'full' consumption is a constant fraction δ of total wealth. From (3c), the instantaneous rate of growth of consumption equals the excess of the real interest rate over the subjective rate of time preference. The marginal propensity to consume is age-independent, reflecting the fact that remaining lifetime is infinite irrespective of the age of the dynasty. Because of the assumption that preferences are logarithmic and separable in consumption and real balances, the marginal propensity to consume out of wealth is constant and the rate of growth of individual consumption is independent of the nominal interest rate.[5] Eq. (3d) is the real money demand function: optimal money holdings are increasing in consumption and decreasing in the nominal interest rate.

Define aggregate consumption per capita, $C(t)$, as

$$C(t) = \left\{ N(0)c(0,t) + \int_0^t c(s,t)\,\mathrm{d}N(s) \right\} \Big/ N(t). \tag{4}$$

Similar definitions hold for other variables, with upper-case letters denoting the aggregate per capita counterparts to lower-case magnitudes [notice that this definition implies that $M(t)$ denotes per capita aggregate *real, not nominal*, money balances].

From (3b) and (3d) we have, dropping time arguments when no confusion results,

$$C = \delta(1-\mu)[A+H], \tag{5a}$$

$$M = [\mu/(1-\mu)][C/i]. \tag{5b}$$

Aggregate consumption is linear in total wealth $A+H$. (5b) is the aggregate real money demand function.

The laws of motion of aggregate non-human and human wealth are easily deduced, using (5), from (3a), (3e) and (4),

$$\mathrm{d}A/\mathrm{d}t = (r-n)A + Y - C/(1-\mu), \tag{5c}$$

$$\mathrm{d}H/\mathrm{d}t = rH - Y, \tag{5d}$$

[5]This causality has been noted by Cohen (1985), in his study of Fischer's (1979) article.

as

$$Y(t) = y(t) \quad \text{and} \quad H(t) = h(t),$$

because of the assumption made on non-interest income profiles. Per capita aggregate non-human and human wealth do not accumulate at the same rate, unless $n = 0$. This crucial discrepancy arises from the fact that while new families are born, by construction, with zero non-human wealth they arrive into the world with the same human wealth as older families. Notice that we must also have, from (3e),

$$\lim_{\theta \to \infty} H(t+\theta)\exp\left\{-\int_t^{t+\theta} r(x)\,\mathrm{d}x\right\} = 0, \tag{5e}$$

a condition which will be used *infra*.

Using (5), aggregate consumption evolves according to

$$\mathrm{d}C/\mathrm{d}t = (r-\delta)C - n\delta(1-\mu)A, \tag{6}$$

which is the central equation of this model, indicating that, unless $n = 0$, individual and per capita aggregate consumption do not follow the same law of motion. Together with the initial condition (2e), eqs. (4) and (5) fully characterize the aggregate consumption optimum.

2.2. Technology

Output is manna from heaven, non-produced and non-storable. Every consumer alive receives, at any given instant of time, a time and age independent endowment $e > 0$ of this consumption good.

2.3. Government

Let $G(t)$ denote real per capita government consumption, and $T(t)$ the age-independent lump-sum head tax (transfer if negative) levied by the government on every individual alive at time t, so that disposable non-interest income is simply

$$y(t) = e - T(t). \tag{7}$$

Define $D(t) = G(t) - T(t)$ as being the government *primary* deficit, i.e., net of interest payments on the national debt. The deficit (surplus if negative) must be financed by selling bonds[6] to the public and/or by creating money (an activity in which monetary authorities are assumed to have a legal monopoly). Letting S denote per capita money seigniorage, the government instantaneous budget constraint is therefore given by

$$D = \{(n-r)B + \mathrm{d}B/\mathrm{d}t\} + S, \tag{8a}$$

$$B(0) \quad \text{given}.$$

The term $(n-r)B + \mathrm{d}B/\mathrm{d}t$ represents per capita bond seigniorage, while, by definition of aggregate per capita real balances, money seigniorage satisfies

$$S = (n + \Pi)M + \mathrm{d}M/\mathrm{d}t. \tag{8b}$$

Following Sargent and Wallace (1981), I specify fiscal and monetary authorities to be separate institutions, with the fiscal authority 'dominating' in that it chooses the level of public consumption and lump-sum taxes independently of the actions of the monetary authority. More specifically I assume, to focus on the effects of *permanent deficits*, that $G(t)$, $T(t)$ and hence $D(t)$ are constant over time, with $D = G - T \geq 0$, $0 \leq G < e$. Given the permanent deficit selected by the fiscal authority, the choice by monetary policymakers of a particular rate of growth of the nominal money supply, denoted by σ, determines the respective contributions of money and bond seigniorage to the financing of the deficit, with per capita money seigniorage given by

$$S = \sigma M. \tag{8c}$$

The question which, in the wake of Sargent and Wallace (1981), I address in section 3 is the extent to which general equilibrium considerations restrict, for a given deficit, the range of rates of money expansion which monetary authorities may choose from.

2.4. Equilibrium

In equilibrium, it must be the case that

$$C + G = e, \tag{9a}$$

$$W = B, \tag{9b}$$

i.e., that the markets for the consumption good and for private, inside credit

[6] These bonds are assumed to be real bonds. In this economy, the substitution of nominal for real bonds would be neutral.

clear (so that, in the aggregate, all non-human wealth is held in the form of money or government bonds: $A = M + W = M + B$). Given G, T and σ, eqs. (5), (6), (7), (8) and (9) fully characterize a perfect foresight equilibrium. Noting that, from (9a), market-clearing per capita aggregate private consumption, C, is constant over time and equal to $e - G$, and using (5b) to eliminate the inflation rate from (8b), the equilibrium dynamics of real balances, public debt and the real interest rate are given, from (6) and (8), by

$$\mathrm{d}C/\mathrm{d}t = (r-\delta)(e-G) - n\delta(1-\mu)(M+B) = 0, \tag{10a}$$

$$\mathrm{d}B/\mathrm{d}t = (r-n)B + D - \sigma M, \tag{10b}$$

$$\mathrm{d}M/\mathrm{d}t = (r-n+\sigma)M - (e-G)\mu(1-\mu)^{-1}, \tag{10c}$$

with $B(0)$ given.

Three other conditions must be satisfied in equilibrium. Firstly, *individual* consumption must be non-negative at every instant, a restriction which requires, from (2f), (3b), (3c) and the fact that per capita aggregate and individual human wealth are by assumption identical, that human wealth, H, be non-negative. From (5a), therefore, it must be the case that $A \leq C/[\delta(1-\mu)]$ or, from (10a), that $r(t) \leq n + \delta$ for all t. Secondly, it can easily be shown that (5e) requires that the interest rate be strictly positive in the long run. Finally, the non-negativity of the price level (i.e., of real balances) constrains, from (5b), the nominal interest rate to be non-negative at every instant.

3. Analysis

I now study which restrictions, if any, are placed by general equilibrium considerations upon the choice by monetary policymakers of a rate of growth of nominal money supply, *given the path of government spending and lump-sum taxes* independently selected by the 'dominating' fiscal authorities. I thus re-examine the debate between Sargent and Wallace (1981), Darby (1984), McCallum (1984), Miller and Sargent (1984), Liviatan (1984), and Drazen (1984). I first briefly analyze the special case in which no intergenerational effects are present ($n = 0$). I then show that the presence of intergenerational effects, as measured by n, enlarges the set of admissible monetary policies.

3.1. No intergenerational effects

When n is equal to zero, eq. (10a) indicates that the equilibrium real interest rate is equal, *at every instant*, to the rate of time preference, δ, independently of the fiscal/monetary policy pursued. This is of course a manifestation of the fact that, when $n = 0$, this model reduces to Sidrauski's (1967), and of the assumptions on technology and tastes.[7] The laws of motion of B and M can then be rewritten as

$$\mathrm{d}B/\mathrm{d}t = \delta(B - B^*) - \sigma(M - M^*), \tag{11a}$$

$$\mathrm{d}M/\mathrm{d}t = (\delta + \sigma)(M - M^*), \tag{11b}$$

where

$$M^* = (e - G)\mu(1-\mu)^{-1}(\delta+\sigma)^{-1} \quad \text{and} \quad B^* = (\sigma M^* - D)/\delta,$$

denote steady-state real balances and debt, respectively.[8] The triangular system of linear differential equations (11) has two eigenvalues, δ and $\delta + \sigma$, which are both positive under the equilibrium restriction that long-run real balances be positive, i.e., $\delta + \sigma > 0$. The steady-state equilibrium (B^*, M^*), when it exists, is thus unstable. It can be shown that paths which do not coincide with the steady state at every instant are not equilibrium trajectories, as they result in violations of either non-negativity or transversality conditions.

Given the initial stock of public debt $B(0)$ and the levels of government consumption and lump-sum taxes, G and T, independently chosen by fiscal authorities, the only choice of a rate of growth of the nominal money supply, σ, which is consistent with general equilibrium is therefore the one which makes $B(0)$ a steady state. It must thus be the case that $B^* = B(0)$, or, equivalently, that

$$\sigma = -\delta\left\{[\delta B(0) + D]\Big/\left[\delta B(0) + D - (e - G)\mu(1-\mu)^{-1}\right]\right\} \equiv \sigma^*. \tag{12a}$$

This is a valid equilibrium, with a positive long-run nominal interest rate $\delta + \sigma^*$, if and only if the *consolidated* deficit $\delta B(0) + D$ does not exceed $(e - G)\mu(1-\mu)^{-1}$. This condition suffices to ensure, when $B(0)$ is positive, that $\sigma^* > 0$. With a positive long-run interest rate (δ) and positive initial debt, a permanent primary deficit must therefore be accompanied by money creation, as shown by McCallum (1984).

More interestingly, under our assumptions,

$$\mathrm{d}\sigma^*/\mathrm{d}D|_{\mathrm{d}G=0} > 0, \tag{12b}$$

[7]With $n = 0$ and under these assumptions, the model analyzed here is analogous with the one studied by Liviatan (1984). Drazen (1984) and Drazen and Helpman (1986) examine the implications for aggregate dynamics of more general, but still *separable* utility functions while remaining within the Sidrauski (1967) framework.

[8]$\mathrm{d}M^*/\mathrm{d}\sigma = -\mathrm{d}B^*/\mathrm{d}\sigma = -M^*/(\delta+\sigma) > 0$.

which establishes that any (tax-cut induced) increase in the permanent primary deficit must be entirely monetized. This is of course because steady state bond seigniorage is fixed at $-\delta B(0)$ so that, from (8a), any additional permanent deficit must be financed by money seigniorage, and because, as can be seen from (10c), steady state money seigniorage σM^* unambiguously increases with σ when the interest rate net of the growth rate (which here is simply δ) is positive.

The foregoing discussion thus confirms that, in a model à la Sidrauski (1967) with no intergenerational effects, monetary authorities have no freedom to select the rate of growth of the money supply in the face of an exogenously fixed permanent primary budget deficit, as argued by McCallum (1984). I now show that this conclusion does not generalize to an economy in which intergenerational effects are present and in which the long-run real interest rate is *not* invariant to the size of the permanent primary deficit.

3.2. Intergenerational effects

When new and economically disconnected cohorts enter the economy over time (i.e., when n is strictly positive), it is easy to see, by adding (10b) to (10c), that the equilibrium dynamics of the real interest rate and total non-human wealth, $A = M + B$, are independent of the rate of growth of the nominal money supply,[9] σ, but *not of fiscal policy*, as summarized by G and T. This is most easily seen by differentiating (10a) with respect to time, which, after some substitutions, yields the equilibrium law of motion of the real interest rate:

$$\mathrm{d}r/\mathrm{d}t = r^2 - r(n+\delta) + n\delta(1-\mu)[1 + D/(e-G)] \equiv Q(r). \quad (13a)$$

Inspection of the polynomial $Q(r)$ reveals that, provided that D does not exceed a maximum level $\bar{D} \equiv (e-G)\{[n+\delta]^2/[4n\delta(1-\mu)] - 1\} > 0$ (a condition which I will henceforth assume to be satisfied), the equation $Q(r) = 0$ has two real roots, which are both positive. There are thus, for each fiscal policy (G, T), two steady-state real interest rates, the smallest of which, r_1, is stable, while the largest one, r_2, is unstable – a familiar phenomenon. The response of those steady-state interest rates to a (tax-cut induced) increase in the primary deficit D is easily computed to be

$$\mathrm{d}r_i/\mathrm{d}D|_{\mathrm{d}G=0} = -n\delta(1-\mu)/\{[2r_i - (n+\delta)][e-G]\}, \quad (13b)$$

$$i = 1, 2,$$

[9] This independence is characteristic in steady state of the class of utility functions $u(c, m) = (c^{1-\mu}m^{\mu})^{1-\alpha}/(1-\alpha)$, $\alpha > 0$, analyzed by Fischer (1979). It carries over to the transition path only in the case $\alpha = 1$, i.e., in the logarithmic case studied here.

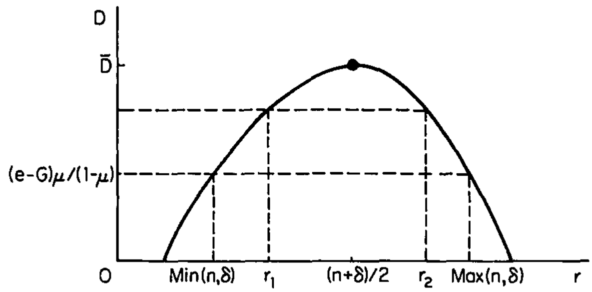

Fig. 1. Determination of the steady state interest rates.

so that, given $r_1 + r_2 = (n+\delta)$ and $r_1 < r_2$, the stable root r_1 increases while the largest root r_2 decreases with the deficit. When $D = \bar{D}$, $r_1 = r_2 = (n+\delta)/2$. One can also check that when $D = (e-G)\mu/(1-\mu) < \bar{D}$, the two stationary interest rates are simply $r_1 = \min(n, \delta)$ and $r_2 = \max(n, \delta)$. Moreover, $r_1 \to 0^+$ and $r_2 \to \delta$ as $n \to 0$; when n is strictly zero, however, the $r_1 = 0$ root does *not* constitute an equilibrium anymore, as it would violate (5e), so that only the time-preference interest rate equilibrium, $r_2 = \delta$, remains.[10] Finally, as required by non-negativity constraints on consumption, both r_1 and r_2 are below the critical value $n+\delta$, as $Q(n+\delta) > 0$ and $Q'(n+\delta) > 0$. The foregoing properties are summarized in fig. 1

Let M_i and B_i denote, respectively, the steady-state per capita aggregate real balances and stock of public debt associated with the interest rate r_i, $i = 1, 2$. From (10), we have

$$M_i = \left[\mu(1-\mu)^{-1}(e-G)\right] \Big/ (r_i - n + \sigma), \tag{14a}$$

$$B_i = [\sigma M_i - D] / (r_i - n). \tag{14b}$$

Notice that, although r_i itself does not depend on σ, both M_i and B_i do, and that steady-state seigniorage, σM_i, increases or decreases with σ depending on whether the interest rate r_i exceeds or falls short of the population growth rate n.

[10]Intuitively, if the $r_1 = 0$ root were an equilibrium in the representative family case ($n = 0$), individual and thus aggregate consumption decline over time at rate δ, which would be inconsistent with market clearing at the constant $e - G$. For $n = \varepsilon > 0$, ε very small, however, r_1 is close to zero but positive, and individual consumption declines at a rate approximately equal to δ – which is not *a priori* inconsistent with per capita aggregate consumption remaining constant at $e - G$ since the large consumption of few relatively young dynasties is matched by the small consumption of the numerous relatively old.

The local stability properties of the steady states are characterized by linearizing (10) around $E_i = (r_i, M_i, B_i)$:

$$\begin{bmatrix} \mathrm{d}r/\mathrm{d}t \\ \mathrm{d}M/\mathrm{d}t \\ \mathrm{d}B/\mathrm{d}t \end{bmatrix} = \begin{bmatrix} 2r_i - (n+\delta) & 0 & 0 \\ M_i & r_i - n + \sigma & 0 \\ B_i - \dfrac{\sigma(e-G)}{n\delta(1-\mu)} & 0 & r_i - n + \sigma \end{bmatrix} \begin{bmatrix} r - r_i \\ M - M_i \\ B - B_i \end{bmatrix}. \tag{15}$$

Given the triangular nature of this system, its eigenvalues are simply the diagonal elements of the above matrix. The first eigenvalue, $2r_i - (n+\delta)$, is unambiguously negative for $i = 1$, positive for $i = 2$ (as $r_1 + r_2 = n + \delta$ and $r_1 < r_2$). The other two eigenvalues have the sign of $r_i - n + \sigma$, which is simply the nominal interest rate at E_i. It is therefore necessary to distinguish between three cases (neglecting boundary situations for brevity):

3.2.1. Case 1

If $\sigma < n - r_2 < n - r_1$, then neither E_1 nor E_2 is a feasible steady-state equilibrium as each is associated with a negative nominal interest rate and thus, from (14a), with negative real balances. Moreover, E_1 is stable (three negative eigenvalues), while E_2 is saddlepoint stable (one positive and two negative eigenvalues). To avoid a negative asymptotic nominal interest rate, the economy would therefore need to find itself on the unstable two-dimensional manyfold corresponding to the *positive* eigenvalue at E_2 [which, given the single predetermined variable $B(0)$, would yield a unique solution]. It can be shown that this diverging path would not constitute, however, a valid equilibrium, as it would result in either real balances or the consumption of the newly born dynasties becoming negative in finite time. For a given fiscal policy, the condition $\sigma > n - r_2$ is thus necessary for equilibrium.

3.2.2. Case 2

Suppose now that, given G and T, monetary authorities indeed choose a rate of money growth such that $\sigma > n - r_2$, but that they intend to keep σ below $n - r_1$. For σ in this range, only E_2 is a valid steady-state equilibrium, with positive nominal interest rate.[11] It is however a source (three positive eigenval-

[11] In the case, studied in section 3.1, in which n is strictly equal to zero, the E_1 steady state is ruled out not because $\sigma < n - r_1 = 0$, but because $r = r_1 = 0$ violates (5e). See also footnote 10 on this point.

ues), while E_1 is stable (three negative eigenvalues). Given the initial stock of public debt, an equilibrium will therefore exist if and only if, for the given fiscal policy (G, T), there exists a $\hat{\sigma}$ in the interval $(n - r_2, n - r_1)$ such that $B_2 = B(0)$. From (8a) or (14b), the unique rate of growth of the nominal money supply, $\hat{\sigma}$, which equalizes B_2 to $B(0)$, is the solution to

$$S[\sigma, D] = \Delta[B(0), D], \tag{16a}$$

an equation which simply states that steady state money seigniorage,

$$S[\sigma, D] \equiv \left[\mu(1-\mu)^{-1}(e-G)\right]\left[\sigma/(r_2(D) - n + \sigma)\right],$$

must be equal to the steady state consolidated budget deficit, denoted by

$$\Delta[B(0), D] \equiv D + [r_2(D) - n]B(0)$$

[taking into account the equilibrium dependence of r_2 on D established in (13)]. I now examine the response of $\hat{\sigma}$ to (tax-cut induced) increases in the primary deficit, and then suggest that this discussion is not vacuous, i.e., that $\hat{\sigma}$ is in the interval $(n - r_2, n - r_1)$.

Consider first the impact on $\hat{\sigma}$ of a (tax-cut induced) increase in D. Differentiating (16a) yields, for a given $B(0)$,

$$\mathrm{d}\hat{\sigma}/\mathrm{d}D|_{\mathrm{d}G=0} = (\Delta_2 - S_2)/S_1, \tag{16b}$$

an expression whose sign is in general ambiguous.[12] Suppose, for an instant, that the long-run real interest rate were invariant with respect to D (so that $\Delta_2 = 1$ and $S_2 = 0$), as in the overlapping-generations models of Sargent and Wallace (1981) and Darby (1984) or the Sidrauski (1967) framework used by McCallum (1984), Liviatan (1984) and Drazen (1984). The 'simple arithmetics' of the government budget constraint (16) would then imply that the response of $\hat{\sigma}$ to increases in the primary deficit would solely depend on the sign of S_1, i.e., of $r_2 - n$. These simple arithmetics are misleading, however, as soon as the long-run real interest rate r_2 depends, as it does in this model, on D. The response of $\hat{\sigma}$ to an increase in D is in general ambiguous, because changes in D affect *both* the steady state consolidated deficit (through the effect on bond seigniorage) and steady state money seigniorage. The importance of the first effect has been emphasized by Miller and Sargent (1984) in their response to Darby (1984). The second effect (absent of their paper because of their

[12] S_1 and Δ_1 have the sign of $r_2 - n$; S_2 has the sign of, and is proportional to $-r_2'(D)$, while $\Delta_2 = 1 + B(0)[r_2'(D)]$ is, when $B(0) > 0$, positive at first and then negative because of the negative effect of D on r_2. From (13b), the maximum consolidated deficit is reached when D is such that $r_2 = (n+\delta)/2 + \frac{1}{2}n\delta(1-\mu)[B(0)/(e-G)] > (n+\delta)/2$.

specification of a quantity theory demand for money) is likely to be present in any fully specified general equilibrium model; its existence reinforces the conclusion that simple comparisons between the real interest rate and the growth rate are in general insufficient to predict whether higher permanent deficits must be monetized or not. Moreover, as I show below, the class of admissible equilibria is much wider than the one under consideration here, which compounds the ambiguity.

To verify, finally, that this discussion is not vacuous, consider the special case in which $B(0)$ is zero[13] – an assumption which eliminates the effects of changes in D on bond seigniorage. Using (16a), the conditions under which the only equilibrium is E_2 are simply

$$\hat{\sigma} = n\delta\mu/(r_2 - \delta) + n - r_2, \tag{17a}$$

$$\hat{\sigma} > n - r_2, \tag{17b}$$

$$\hat{\sigma} < n - r_1 = r_2 - \delta. \tag{17c}$$

There is a non-empty set of points $(r_2, \hat{\sigma})$ which satisfy these inequalities. It is characterized by $r_2 > \hat{r} > (n+\delta)/2$, where $\hat{r}$ is the unique solution larger than δ to (17a) and (17c) with equality. Similar arguments can be applied when the initial stock of debt is positive.

3.2.3. Case 3

If $\sigma > n - r_1 > n - r_2$, then both E_1 and E_2 are admissible equilibria, with positive nominal interest rates. E_1 is saddlepath stable (one negative, two positive eigenvalues), while E_2 is a source (three positive eigenvalues). Given that there is a single predetermined variable in this economy, the initial stock of bonds $B(0)$, and that the stable manyfold associated with E_1 has dimension 2, there exist a unique choice of initial real interest rate and real balances leading, for $B(0)$ close to B_1, to E_1.[14] It can easily be shown, using the results of section 2.4, that all other trajectories [but for, of course, the steady state E_2 if $B(0)$ equals B_2] are not equilibrium paths, as they result in non-negativity constraints on either individual consumption or the price level being violated in finite time.

As there is no *a priori* reason for the initial stock of debt $B(0)$ to be confined within the neighborhood of either steady state, it is necessary to study the global dynamics of this economy. This is done in fig. 2, which exploits the recursivity of (10) and (13) to draw separate phase diagrams in

[13] A more general study of existence issues is omitted, for brevity, from this study.

[14] See Blanchard and Kahn (1980).

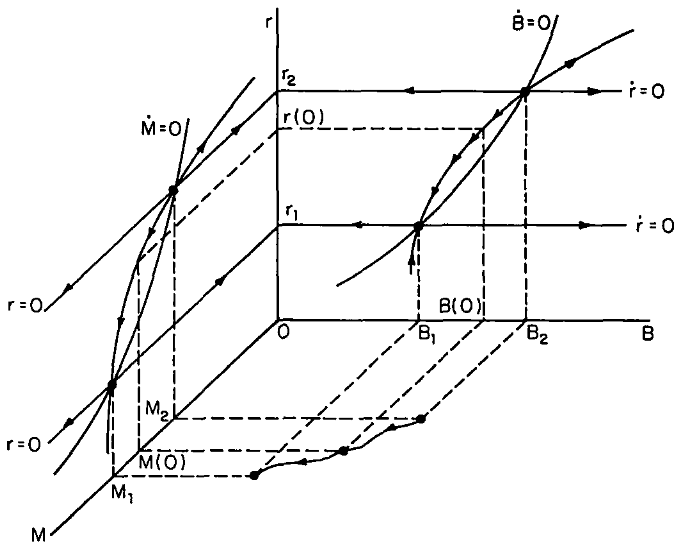

Fig. 2. Equilibrium dynamics.

(r, B) and (r, M) spaces.[15] From this figure, it is immediate that given $\sigma > n - r_1$, a necessary and sufficient condition for the existence of an equilibrium of the type described here is that the initial stock of debt, $B(0)$, does not exceed B_2, the stock of debt corresponding to the high interest rate steady state. Using (14) to rewrite the inequality $B(0) \leq B_2$, the range of options open to monetary authorities, when they are faced with a given fiscal policy (G, T) and a given $B(0)$, is therefore described by

$$[D + (r_2 - n)B(0)][1 - \sigma/\hat{\sigma}] \leq 0, \tag{18a}$$

$$\sigma > n - r_1 = r_2 - \delta. \tag{18b}$$

When (18a) is satisfied with equality (which occurs if monetary authorities select $\hat{\sigma}$ as the rate of growth of the nominal money supply), the economy remains forever in steady state at E_2, very much as in case 2 *supra*. If,

[15]The lines $r = r_1$ and $r = r_2$ constitute the $\mathrm{d}r/\mathrm{d}t = 0$ locus. The $\mathrm{d}M/\mathrm{d}t = 0$ locus is, from (10c), downward sloping in the (r, M) space. The slope of $\mathrm{d}B/\mathrm{d}t = 0$ locus can be shown to have the same sign as $v \equiv \sigma(\sigma + \delta - n) + n\delta(1 - \mu)D/(e - G)$. But, by assumption, $r_2 > r_1 > n - \sigma$, so that we must have, from (11), $Q(n - \sigma) = v - n\delta\mu > 0$. Hence $v > n\delta\mu > 0$, so that the $\mathrm{d}B/\mathrm{d}t = 0$ locus is upward sloping in (r, B) space.

however, (18a) is satisfied with strict inequality, the unique saddlepath trajectory will asymptotically converge towards the low interest rate steady state, E_1. As long as $D+[r_2-n]B(0)$ is positive (i.e., as long as choosing $\sigma=\hat{\sigma}$ would entail a positive consolidated deficit), paths which converge to E_1 are eventually more inflationary than those remaining at E_2. But if the stable interest rate r_2 is below n while the initial stock of national debt is very large relative to the size of the primary deficit (so that there is a consolidated budget *surplus*), paths which converge to E_1 are less inflationary than those which remain at E_2.

The analysis of existence issues is again limited to the case in which $B(0)=0$. Existence conditions when $D>0$ and $B(0)=0$ are simply, using (17a) and (18),

$$\sigma \geq \hat{\sigma} = n\delta\mu/(r_2-\delta) + n - r_2, \tag{19a}$$

$$\sigma > n - r_1 = r_2 - \delta. \tag{19b}$$

The non-empty set of points satisfying these inequalities is characterized by $\sigma \geq \hat{\sigma}$ for $(n+\delta)/2 \leq r < \hat{r}$ and $\sigma > r_2-\delta$ for $r_2 \geq \hat{r}$, where $\hat{r}$ is defined *supra*.

3.2.4. Discussion

I now use the preceding results to determine whether the presence of intergenerational effects decreases the need for monetization of the permanent deficit. The simplest (but not most complete) way to address this issue is to compare σ^*, the required rate of monetary growth in the absence of intergenerational effects, with $\hat{\sigma}$. By definition of these two magnitudes, it must be the case that

$$\delta B(0) + D - \sigma^* M^* = 0, \tag{20a}$$

$$(r_2-n)B(0) + D - \hat{\sigma}M_2 = 0. \tag{20b}$$

Now, in equilibrium, $r_2 < n+\delta$, as shown above. Therefore, if $B(0)>0$, $\sigma^*M^* < \hat{\sigma}M_2$. Using the definitions of M^* and M_2, this implies that

$$\hat{\sigma}/(r_2-n+\hat{\sigma}) < \sigma^*/(\delta+\sigma^*). \tag{21a}$$

Equilibrium conditions impose that $r_2-n+\hat{\sigma}>0$ and $\sigma^*>0$ [see (12a)]. Therefore, $\hat{\sigma}(\delta+\sigma^*) < \sigma^*(r_2-n+\hat{\sigma})$, which implies that $\hat{\sigma}\delta < \sigma^*(r_2-n) <$

$\sigma^*\delta$, and thus that

$$\hat{\sigma} < \sigma^*, \tag{21b}$$

which suggests that the minimum rate of money creation required to finance a permanent deficit is likely to be smaller when n is positive. Heuristically, the arrival of new cohorts makes it possible for a larger share of the permanent deficit to be financed by bond creation as agents alive today do not have to absorb all the future bond issues associated with the permanent primary deficit [which, as shown by McCallum (1984), they would refuse to do in a consumption optimum].

4. Conclusion

The Sargent–Wallace issue of whether permanent primary budget deficits eventually have to be monetized has been re-examined within the context of a monetary model, hybrid of the Sidrauski and overlapping-generations frameworks, in which the magnitude of intergenerational effects can be simply parametrized by the speed of arrival of new infinitely-lived cohorts.

It has been shown that, given the levels of government spending and lump-sum taxes independently set by fiscal authorities, the set of admissible monetary policies is likely to be much wider as soon as the economy departs from the representative agent framework and the real interest rate is not fixed in the long run.

Simple comparisons between the long-run real interest rate and the economy's growth rate, such are those performed by Sargent and Wallace (1981) and Darby (1984), are in general not sufficient to determine whether higher deficits are accompanied by higher rates of money creation. This is due not only to the existence of a bond revenue Laffer curve, as documented in Miller and Sargent (1984), but also, as demonstrated in this model, to the presence of a money seigniorage Laffer curve.

Finally, the fact that the agents alive today do not have, when new cohorts enter the economy over time, to absorb all the future public debt issues associated with the deficit makes it possible, in most cases, for monetary authorities to monetize a smaller fraction of any given permanent deficit.

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